

Simultaneous Bidirectional Link Selection in Full Duplex MIMO Systems

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Abstract

In this paper, we consider a point to point full duplex (FD) MIMO communication system. We assume that each node is equipped with an arbitrary number of antennas which can be used for transmission or reception. With FD radios, bidirectional information exchange between two nodes can be achieved at the same time. In this paper we design bidirectional link selection schemes by selecting a pair of transmit and receive antenna at both ends for communications in each direction to maximize the weighted sum rate or minimize the weighted sum symbol error rate (SER). The optimal selection schemes require exhaustive search, so they are highly complex. To tackle this problem, we propose a Serial-Max selection algorithm, which approaches the exhaustive search methods with much lower complexity. In the Serial-Max method, the antenna pairs with maximum “obtainable SINR” at both ends are selected in a two-step serial way. The performance of the proposed Serial-Max method is analyzed, and the closed-form expressions of the average weighted sum rate and the weighted sum SER are derived. The analysis is validated by simulations. Both analytical and simulation results show that as the number of antennas increases, the Serial-Max method approaches the performance of the exhaustive-search schemes in terms of sum rate and sum SER.

Index Terms

bidirectional link selection, full duplex, Serial-Max selection method.

I. INTRODUCTION

Current wireless communications systems typically exploit half duplex (HD) transmission. This is because for many years the full duplex (FD) transmission has been considered impractical. The signal leakage from the local output to input in the FD radio, referred to as the self interference, may overwhelm the receiver, thus making it impossible to extract the desired signals. Very recently, there has been a significant process in self-interference suppression in FD radios. The passive suppression methods [1]–[8] design the antennas using a combination of path loss, cross polarization and antenna directionality, while the active approaches [1], [3], [6], [8]–[10] exploit the knowledge of self interference in cancelation in the analog or digital domain. The residual interference still exists and can be modeled as Rayleigh fading [3], [16], [17] when the direct link is effectively suppressed.

These suppression techniques can significantly reduce the self-interference, which has made FD radios practically feasible in the near future. This significant progress in FD has recently inspired some very interesting work on FD signal processing. In [8], [11]–[15], the theoretical limits of point to point bidirectional FD have been investigated by taking into account the residual self-interference after suppression. In [8], the authors derived lower and upper bounds of the achievable sum rate of bidirectional FD communications, and proposed a transmission scheme to maximize the lower bound. In [11], the achievable sum rate of bidirectional FD MIMO systems was analyzed and compared to the conventional HD MIMO systems over a spatial correlated channel. The ergodic capacity of bidirectional FD transmission using one transmit antenna and multiple receive antennas in the presence of channel estimation error has been derived in [12]. a FD antenna mode selection scheme was investigated in [13] for a simple 2×2 MIMO system, where each antenna is either configured as the transmit or receive antenna mode. In [14], [15], the suboptimal and optimal dynamic power allocation schemes were developed based on the sum rate maximization criterion.

In this paper, we consider a general FD MIMO system with N_A and N_B antennas equipped at two nodes. Such an FD MIMO system will create $N_A N_B$ possible links between the two FD MIMO nodes, with one possible link representing the channel from a transmit antenna of a node to a receive antenna of the other node. Since FD radios enable simultaneous bidirectional

information exchanges between two FD MIMO nodes, a fundamental question arisen in such a system is how to select the link for each direction to optimize the system performance. In this paper we consider two performance metrics, weighted sum rate maximization and weighted sum symbol error rate (SER) minimization. The optimal¹ approach requires the exhaustive search from all possible antenna links, however, as the number of antennas increases, such a brute-force search bears very high complexity in selection process.

To resolve this issue, in this paper we propose a simple Serial-Max selection algorithm by selecting the link with optimal performance for each direction in a two-step serial way, which can achieve asymptotically optimal performance. By using the law of total probability and order statistics, the probability distribution functions of the two selected links are calculated, based on which, the closed-form expressions on average weighted sum rate and sum SER are derived. We show that the Serial-Max method approaches the brute-force search method in terms of the average weighted sum rate and sum SER as the number of antennas increases. The theoretical results are verified by Monte-Carlo simulations.

The rest of this paper is organized as follows: Section II introduces the system model. The proposed Serial-Max selection algorithm is presented in Section III. Section IV analyzes the performance of the Serial-Max method, including the average weighted sum rate and sum SER. Simulation results are provided in Section V. In Section VI, we draw the main conclusions.

II. SYSTEM MODEL

In this paper, we consider a bidirectional communication scenario between a pair of FD transceivers, node A and B , as illustrated in Fig. 1, where node A and B are equipped with N_A and N_B antennas, respectively. Both nodes use the same frequency band at the same time for FD operation. Each node employs only one transmit and one receive RF chains, and any antenna can be configured to connect either the transmit or receive RF chain. In the proposed simultaneous bidirectional link selection (SBLS) scheme, two antenna links are selected for simultaneous bidirectional communication by selecting a pair of transmit and receive antennas at both ends. Within each antenna pair, one antenna is selected for transmission and one is for reception.

¹“Optimal” in this paper means that this scheme can achieve the optimal performance under practical constraint, that only the distribution of the residual interference rather than the instantaneous one can be obtained at each node.

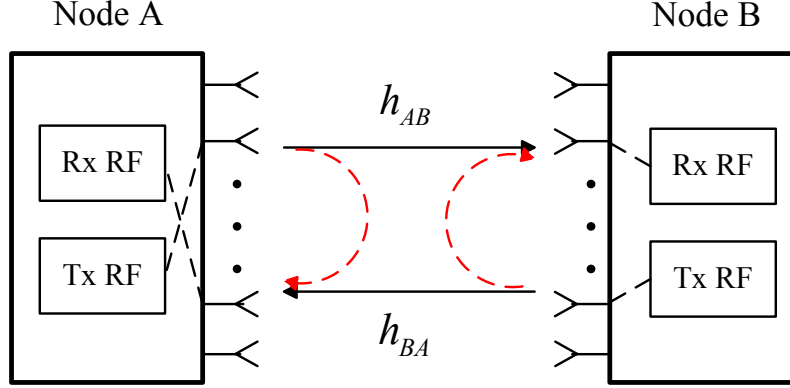


Fig. 1. Full duplex MIMO systems with simultaneous bidirectional link selection

We assume that the links between the two nodes are reciprocal and subject to independent Rayleigh fading, and together denoted by a $N_A \times N_B$ channel matrix $\mathbf{H} = [h^{ij}]$, $i = 1, 2, \dots, N_A, j = 1, 2, \dots, N_B$. The entry h^{ij} represents the fading coefficient from the i -th antenna at node A to the j -th antenna at node B , and it follows the circularly symmetric Gaussian distribution $\mathcal{CN}(0, \sigma_h^2)$. All the possible communication channels are assumed to follow the non-selective independent block fading, where the channel coefficients remain constant during a time slot, and vary from one to another independently. In the beginning of each time slot, all the possible communication links can be estimated perfectly.

Since the two nodes are operated in the FD mode, there exists self interference caused by the signal leakage from the transmit antenna to the receive antenna at the same node. We assume the passive propagation suppression and active analog/digital cancelation techniques are employed to cancel the self interference. As indicated in [16], [17], the direct link of the self interference can be effectively suppressed, and the residual self interference can be approximated to follow the Rayleigh distribution [20], [21]. In this paper, due to practical constraints, we assume that only the distribution of the residual interference, including the mean and variance, can be obtained and used in the selection process. Therefore, the selection process is based on the “obtainable SINR”. This is equivalent to that based on the instantaneous SNR, which will be later shown that the former is a scaled version of the latter.

In the SBLS, the bidirectional links, i.e., (I_T, J_R) , from the I_T -th antenna at node A to the J_R -th antenna at node B , and (J_T, I_R) , from the J_T -th antenna at node B to the I_R -th antenna

at node A , are selected. The received signals at node A and B , denoted by y_A and y_B , can be expressed as

$$\begin{aligned} y_A &= \sqrt{P_t} h^{J_T I_R} x_B + \sqrt{P_t} h_A^{RI} x_A + n_A \\ y_B &= \sqrt{P_t} h^{I_T J_R} x_A + \sqrt{P_t} h_B^{RI} x_B + n_B, \end{aligned} \quad (1)$$

where $h^{I_T J_R}$ and $h^{J_T I_R}$ denote the links corresponding to the selected antenna pair from node A to B and that from node B to A . P_t is the transmit power at each node. The second term denotes the residual interference, at nodes A and B , respectively. We assume that both residual interference links, h_A^{RI} and h_B^{RI} , are subject to Rayleigh fading with zero mean and variance σ_{RI}^2 . The AWGN at nodes A and B are denoted by n_A and n_B , which both follow $\mathcal{CN}(0, \sigma_n^2)$.

Without the knowledge of the instantaneous residual interference, the selection procedure is based on the “obtainable SINR” matrix $\Gamma = [\gamma^{ij}]$. The entry in the “obtainable SINR” matrix is defined as $\gamma^{ij} = \frac{\gamma_s^{ij}}{(\lambda_i + 1)}$, where $\gamma_s^{ij} = \frac{P_t |h^{ij}|^2}{\sigma_n^2}$ is the instantaneous SNR, and λ_i is the average INR. γ_s^{ij} follows an exponential distribution with mean $\lambda_s = \frac{P_t \sigma_h^2}{\sigma_n^2}$, and the average INR is given by $\lambda_i = \eta \lambda_s = \frac{P_t \sigma_{RI}^2}{\sigma_n^2}$, where η denotes the cancelation ability. The “obtainable SINR” γ^{ij} is a scaled version of the instantaneous SNR γ_s^{ij} , because we assume that all the links have the same average INR λ_i .

In the following analysis in Section IV, we first calculate the instantaneous performance based on the instantaneous residual interference, and then average it with the distribution of fading channel and residual interference channel. We define the instantaneous SINR $\gamma_{AB} = \frac{\gamma_s^{I_T J_R}}{\gamma_B^{RI} + 1}$ and $\gamma_{BA} = \frac{\gamma_s^{J_T I_R}}{\gamma_A^{RI} + 1}$ for the two selected links. where the instantaneous INR γ_A^{RI} and γ_B^{RI} are both exponential random variables with mean λ_i .

III. SIMULTANEOUS BIDIRECTIONAL LINK SELECTION (SBLS)

In this section, we first introduce two optimal SBLS approaches to maximize the weighted sum rate and minimize the weighted sum SER, respectively, based on the “obtainable SINR” matrix, or equivalently the SNR matrix. Then, a low-complexity method which achieves asymptotically optimal performance, referred to as Serial-Max method, is proposed.

A. SBLS based on Weighted Sum Rate Maximization Criterion (Max-WSR)

In this subsection, we describe the SBLS based on Weighted Sum Rate Maximization criterion (Max-WSR) under the Gaussian input assumption. In this criterion, two communication links

$\{(I_T, J_R), (I_R, J_T)\}$ from the I_T -th transmit antenna at node A to the J_R -th receive antenna at node B and the J_T -th transmit antenna at node B to the I_R -th receive antenna at node A , are selected to maximize the weighted bidirectional sum rate

$$\{(I_T, J_R), (I_R, J_T)\} = \arg \max_{\substack{1 \leq i_t, i_r \leq N_A \\ 1 \leq j_t, j_r \leq N_B \\ i_t \neq i_r, j_t \neq j_r}} \{wR(\gamma^{i_t j_r}) + (1-w)R(\gamma^{i_r j_t})\}, \quad (2)$$

where $R(\gamma) = \log_2 [1 + \gamma]$ denotes the rate under the “obtainable SINR” $\gamma = \frac{\gamma_s}{(\lambda_i + 1)}$, and $0 < w < 1$ is the given weight of the transmission from node A to B , depending on the rate requirement or quality of service (QoS) of each user.

B. SBLS Based on Weighted Sum SER Minimization (Min-WSER)

In the SBLS based on weighted sum SER minimization criterion (Min-WSER) under the assumption that the input signal is modulated with finite constellations, the bidirectional antenna links are selected to minimize the weighted sum SER

$$\{(I_T, J_R), (I_R, J_T)\} = \arg \min_{\substack{1 \leq i_t, i_r \leq N_A \\ 1 \leq j_t, j_r \leq N_B \\ i_t \neq i_r, j_t \neq j_r}} \{wSER(\gamma^{i_r j_t}) + (1-w)SER(\gamma^{i_t j_r})\}, \quad (3)$$

where $SER(\gamma) = \alpha Q(\sqrt{\beta\gamma})$ represents the SER under the “obtainable SINR” $\gamma = \frac{\gamma_s}{(\lambda_i + 1)}$. $Q(\cdot)$ is the Gaussian Q -function [22], and (α, β) is a pair of constants determined by the modulation format, e.g., $\alpha = 1, \beta = 2$ for BPSK modulation.

C. The Proposed Serial-Max SBLS

The aforementioned SBLS schemes for the Max-WSR or Min-WSER criteria both require the brute-force search in order to find the optimal antenna pairs. This will become highly complex in selection process as the number of antennas increases. In this subsection, we introduce a low-complexity selection method, referred to as Serial-Max method, which selects the antenna pairs with maximum “obtainable SINR”, or equivalently, the maximum SNR, in a two-step serial way.

In the first step, the best link with the maximum “obtainable SINR” is selected

$$(I_1, J_1) = \arg \max_{1 \leq i_1 \leq N_A, 1 \leq j_1 \leq N_B} \{\gamma^{i_1 j_1}\}. \quad (4)$$

We use $\gamma^{1st} \triangleq \gamma^{I_1 J_1}$ to denote the “obtainable SINR” of the selected link (I_1, J_1) , and γ_s^{1st} to denote the SNR of this link. By removing the I_1 -th column and J_1 -th row from the “obtainable SINR” matrix Γ (and the corresponding SNR matrix Γ_s), we can obtain a $(N_A - 1) \times (N_B - 1)$ submatrix Γ' (and a corresponding pruned SNR matrix Γ'_s).

In the second step, the link (I_2, J_2) with the maximum SNR is then selected from the pruned submatrix Γ'

$$(I_2, J_2) = \arg \max_{\substack{1 \leq i_2 \leq N_A, 1 \leq j_2 \leq N_B \\ i_2 \neq I_1, j_2 \neq J_1}} \{\gamma^{i_r j_t}\}. \quad (5)$$

We use $\gamma^{2nd} \triangleq \gamma^{I_2 J_2}$ to denote the “obtainable SINR” of the second selected link by the Serial-Max method, which is the maximum element of Γ' , and we use γ_s^{2nd} to denote the SNR of this link, which is the maximum element of the corresponding pruned SNR matrix Γ'_s .

To maximize the weighted sum rate or minimize the weighted sum SER, the time-shared scheme can be employed, which allocates a fraction α of the time to use the best link for node A 's transmission, and the rest $1 - \alpha$ to use the best link for node B 's transmission. Given w , the weighted maximization sum rate problem can be solved by calculating the allocation fraction α . We have

$$R = w [\alpha R(\gamma^{1st}) + (1 - \alpha) R(\gamma^{2nd})] + (1 - w) [\alpha R(\gamma^{2nd}) + (1 - \alpha) R(\gamma^{1st})]. \quad (6)$$

To maximize R , we first calculate the derivative

$$\frac{\partial R}{\partial \alpha} = (2w - 1) [R(\gamma^{1st}) - R(\gamma^{2nd})]. \quad (7)$$

It is obvious that $R(\gamma^{1st}) - R(\gamma^{2nd})$ is positive, therefore the allocation factor α depends on whether $w > 0.5$ or not. Specifically, we have

$$\begin{cases} \alpha = 1, & w > 0.5 \\ \alpha = 0, & w < 0.5 \\ 0 \leq \alpha \leq 1, & w = 0.5 \end{cases}. \quad (8)$$

Then, the weighted sum rate can be rewritten as

$$R = \max(w, 1 - w) R(\gamma^{1st}) + \min(w, 1 - w) R(\gamma^{2nd}), \quad (9)$$

The weighted sum SER minimization problem can be solved in a similar way, and we have

$$SER = \max(w, 1 - w) SER(\gamma^{1st}) + \min(w, 1 - w) SER(\gamma^{2nd}). \quad (10)$$

It is shown from (9) and (10) that the best link selected in the Serial-Max method will be allocated with the greater weight in the weighted sum rate and SER expression.

Regarding the performance of the Serial-Max method, we have the following Lemma.

Lemma 1: The Serial-Max method can achieve the optimal weighted sum rate and sum SER performances simultaneously, if the “obtainable SINR” of the second selected link γ^{2nd} is the second or third largest element of the “obtainable SINR” matrix Γ .

Proof: We use R_i and SER_i to denote the respective rate and SER of the link with the i -th largest “obtainable SINR”. If the second selected link corresponds to the second largest element in the “obtainable SINR” matrix Γ , the weighted sum rate is given by $R = \max(w, 1 - w)R_1 + \min(w, 1 - w)R_2$. It is obvious that in this case the Serial-Max method can achieve the optimal performance in terms of weighted sum rate. Similarly, it can be easily proved that the Serial-Max method can achieve the optimal performance of SER.

Recall that the pruned matrix Γ' is obtained by removing the row and column where the largest element, i.e., the first selected link, is located. Meanwhile, the second selected link is the largest element of the pruned matrix Γ' . Therefore, if the second selected link corresponds to the third largest element in the original “obtainable SINR” matrix, it implies that the second largest element in Γ is removed in the aforementioned manipulation. In other words, the two links associated with the first and second largest elements in Γ share the same antenna, which can not be selected simultaneously. In this case, the largest and third largest elements is the best option for the two selected links. Therefore, the Serial-Max method which selects the largest and third largest elements in Γ achieves the optimal performances of weighted sum rate and sum SER. ■

Then, we have the following proposition.

Proposition 1: The probability that the Serial-Max method does not achieve the same performance as the optimal methods, denoted by P_{not} , is upper bounded by

$$P_{not} \leq \frac{(N_A + N_B - 2)(N_A + N_B - 3)}{(N_A N_B - 1)(N_A N_B - 2)}. \quad (11)$$

Proof: According to Lemma 1, P_{not} is upper bounded by the probability, denoted by $P_{not}^{2,3}$, that the “obtainable SINR” of the second selected link γ^{2nd} is not the second nor third largest elements of the “obtainable SINR” matrix Γ . This implies that both the second and third largest elements are in the same row or column as the largest element, and they are both removed in

the process of obtaining the pruned “obtainable SINR” matrix Γ' . Note that the elements of Γ are independent and identically distributed. Due to symmetry, each element of Γ has the same probability to be the i -th largest element. Thus, we have

$$P_{not}^{2,3} = \frac{(N_A + N_B - 2)(N_A + N_B - 3)}{(N_A N_B - 1)(N_A N_B - 2)}. \quad (12)$$

Combining the fact that P_{not} is no more than $P_{not}^{2,3}$, Proposition 1 can be proved. \blacksquare

It is shown from (11) that the upper bound of P_{not} decreases quadratically as the number of antennas increases, which implies the probability that the Serial-Max method selects the same pairs as the optimal one will increase, and thus approaches the optimal performance in terms of weighted sum rate and sum SER asymptotically.

In addition to the asymptotically-optimal performance, the complexity of the Serial-Max method is much simpler than the exhaustive search approach, as shown in Table I. For the Serial-Max method, in the first step, the maximum “obtainable SINR” is selected from a $N_A \times N_B$ matrix, and $N_A N_B$ comparisons are required. Similarly, for the second step $(N_A - 1)(N_B - 1)$ comparisons are needed, and the Serial method overall needs $2N_A N_B - N_A - N_B + 1$ comparisons. By contrary, the optimal method requires exhaustive search in order to find the optimal antenna pairs, leading to $\frac{N_A N_B (N_A - 1)(N_B - 1)}{2}$ comparisons. Therefore, the proposed Serial-Max algorithm can approach the optimal algorithm with significantly reduced complexity.

TABLE I
COMPLEXITY COMPARISON

	Optimal selection approach	Serial-Max approach
Complexity	$\frac{N_A N_B (N_A - 1)(N_B - 1)}{2}$	$2N_A N_B - N_A - N_B + 1$

IV. PERFORMANCE ANALYSIS

The optimal selection methods are both very difficult to analyze, and thus, in this section we analyze the performance of the Serial-Max algorithm. It will be shown later in simulations that the Serial-Max method can achieve near-optimal performance in terms of average weighted sum rate and SER.

A. Probability Distributions of Two Selected Links

To analyze the performances of the Serial-Max method, the distributions of the real instantaneous SINR corresponding to the two selected links γ_{AB} and γ_{BA} are required. For simplicity and without loss of generality, we consider the case of $w > 0.5$ for the following analysis, which can be easily extended to $w \leq 0.5$.

According to the description of the Serial-Max method in (4), the SNR of the first selected link, γ_s^{1st} is the largest order statistic among $N_A N_B$ i.i.d. exponentially distributed random variables γ_s^{ij} . The corresponding link (I_1, J_1) is used for transmission from node A to B . We have the following Lemma.

Lemma 2: The CDF of γ_{AB} can be given by

$$F_{\gamma_{AB}}(x) = \sum_{k=0}^{N_A N_B} \binom{N_A N_B}{k} (-1)^k \frac{e^{-\frac{k}{\lambda_s} x}}{k\eta x + 1}. \quad (13)$$

Proof: The derivation is given in Appendix A. ■

The second selected link in the Serial-Max method (I_2, J_2) is used for transmission from node B to A . According to (5), we can obtain

Lemma 3: The CDF expression of the instantaneous link SINR of the second selected link, i.e., γ_{BA} , is given by

$$F_{\gamma_{BA}}(x) = \sum_{k=1}^{N_A+N_B-1} \sum_{l=N_A N_B - k}^{N_A N_B} \sum_{m=0}^l (-1)^m \mu_{k,l,m} \frac{e^{-\frac{N_A N_B - l + m}{\lambda_s} x}}{(N_A N_B - l + m) \eta x + 1}, \quad (14)$$

where $\mu_{k,l,m}$ is expressed as

$$\mu_{k,l,m} = \frac{\binom{N_A N_B - k - 1}{N_A + N_B - k - 1} \binom{N_A N_B}{l} \binom{l}{m}}{\binom{N_A N_B - 1}{N_A + N_B - 2}}. \quad (15)$$

Proof: The derivation is given in Appendix B. ■

B. Average weighted Sum Rate

Based on the CDF expressions of the two selected links γ_{AB} and γ_{BA} , in this section, the average weighted sum rate of the two links is obtained. Firstly, the instantaneous rate is calculated for given realizations of the selected communication channel and the corresponding self-interference channel. Then the average rate is obtained by averaging the instantaneous rate

with respect to the distributions of the channels. Under the Gaussian input assumption, the average rate of the link with SINR γ can be obtained as

$$\begin{aligned}\bar{R} &= \mathbb{E} [\log_2(1 + \gamma)] \\ &= \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_\gamma(x)}{1 + x} dx,\end{aligned}\tag{16}$$

where $F_\gamma(x)$ is the CDF of γ .

Proposition 2: The average weighted sum rate of the Serial-Max method, denoted by \bar{R}^{S-Max} is given by

$$\bar{R}^{S-Max} = w\bar{R}_{AB} + (1 - w)\bar{R}_{BA},\tag{17}$$

where \bar{R}_{AB} and \bar{R}_{BA} are the average rates of the two selected links, respectively, which can be expressed as

$$\bar{R}_{AB} = \frac{1}{\ln 2} \sum_{k=0}^{N_A N_B} \frac{(-1)^k \binom{N_A N_B}{k}}{1 - k\eta} \left[e^{\frac{1}{\eta\lambda_s}} \text{E}_1\left(\frac{1}{\eta\lambda_s}\right) - e^{\frac{k}{\lambda_s}} \text{E}_1\left(\frac{k}{\lambda_s}\right) \right],\tag{18}$$

and

$$\bar{R}_{BA} = \frac{1}{\ln 2} \sum_{k=1}^{N_A + N_B - 1} (F_1 + F_2),\tag{19}$$

where

$$F_1 = \sum_{l=N_A N_B - k}^{N_A N_B - 1} \sum_{m=0}^l \frac{(-1)^m \mu_{k,l,m}}{1 - (N_A N_B - l + m)\eta} \left[e^{\frac{1}{\eta\lambda_s}} \text{E}_1\left(\frac{1}{\eta\lambda_s}\right) - e^{\frac{N_A N_B - l + m}{\lambda_s}} \text{E}_1\left(\frac{N_A N_B - l + m}{\lambda_s}\right) \right],\tag{20}$$

and

$$F_2 = \sum_{m=1}^{N_A N_B} \frac{(-1)^{m+1} \mu_{k, N_A N_B, m}}{1 - m\eta} \left[e^{\frac{1}{\eta\lambda_s}} \text{E}_1\left(\frac{1}{\eta\lambda_s}\right) - e^{\frac{m}{\lambda_s}} \text{E}_1\left(\frac{m}{\lambda_s}\right) \right].\tag{21}$$

In addition, $\text{E}_1(\cdot)$ denotes the exponential integral function [22].

Proof: The derivation is given in Appendix C. ■

It is shown from (17)–(21) that the average weighted sum rate of the Serial-Max method \bar{R}^{S-Max} is only determined by the average SNR λ_s and cancelation ability η . When λ_s goes to infinity, we have $\text{E}_1(\varepsilon) \approx -\gamma - \ln(\varepsilon)$ and $e^\varepsilon \approx 1 + \varepsilon$, for $\varepsilon = 1/\lambda_s$, where $\gamma \approx 0.5772$ is Euler's

constant [28]. After some manipulations, \bar{R}^{S-Max} converges to a rate ceiling $\bar{R}_{\lambda \rightarrow \infty}^{S-Max}$

$$\begin{aligned} \bar{R}_{\lambda_s \rightarrow \infty}^{S-Max} \rightarrow & \frac{w}{\ln 2} \sum_{k=0}^{N_A N_B} \frac{(-1)^k \binom{N_A N_B}{k} \ln(k\eta)}{1 - k\eta} + \frac{1-w}{\ln 2} \sum_{k=1}^{N_A+N_B-1} \sum_{m=1}^{N_A N_B} \frac{(-1)^m \mu_{k, N_A N_B, m} \ln(m\eta)}{1 - m\eta} \\ & + \frac{1-w}{\ln 2} \sum_{k=1}^{N_A+N_B-1} \sum_{l=N_A N_B - k}^{N_A N_B - 1} \sum_{m=0}^l \frac{(-1)^m \mu_{k, l, m} \ln[(N_A N_B - l + m)\eta]}{1 - (N_A N_B - l + m)\eta} \end{aligned} \quad (22)$$

C. Average weighted sum SER

In this section, we analyze the average weighted sum SER of the Serial-Max method. For the SER analysis, we assume that the input signal is modulated with a finite constellation. Though the finite constellations like BPSK are used, the current self-interference cancellation technique can reduce the self-interference near to the noise level, as shown in [17], [24]. It has thus been commonly assumed in many existing papers that the residual self-interference after cancellation follow the Rayleigh distributions [16], [20]. We also adopt this assumption in this paper. Firstly, the SER is calculated for a given set of channel realizations, similar to the weighted sum rate analysis. Then, the SER is averaged over the communication and self-interference channels. The average SER of the link with SINR γ can be written as

$$\begin{aligned} \overline{SER} &= \alpha \mathbb{E} \left[Q \left(\sqrt{\beta \gamma} \right) \right] \\ &= \frac{\alpha \sqrt{\beta}}{2\sqrt{2\pi}} \int_0^\infty \frac{F_\gamma(x) e^{-\frac{\beta x}{2}}}{\sqrt{x}} dx. \end{aligned} \quad (23)$$

$Q(\cdot)$ is the Gaussian Q-Function [22].

In addition, if the first-order expansion of the PDF of γ is expressed as

$$f_\gamma(x) = \frac{\zeta x^N}{\lambda^{N+1}} + o(x^{N+\epsilon}), \quad (24)$$

the asymptotic SER can be obtained as [25], [26]

$$\overline{SER} = \frac{2^N \alpha \zeta \Gamma(N + \frac{3}{2})}{\sqrt{\pi} (N+1) (\beta \lambda)^{N+1}} + o\left(\frac{1}{\lambda^{(N+1)}}\right). \quad (25)$$

Proposition 3: The average weighted sum SER of the Serial-Max method, denoted by \overline{SER}^{S-Max} is

$$\overline{SER}^{S-Max} = w \overline{SER}_{AB} + (1-w) \overline{SER}_{BA}, \quad (26)$$

where \overline{SER}_{AB} and \overline{SER}_{BA} are the average SER of the two selected links, and can be calculated as

$$\overline{SER}_{AB} = \frac{1}{2} + \frac{\alpha\sqrt{\beta\pi}}{\sqrt{2}} \sum_{k=1}^{N_A N_B} \frac{(-1)^k \binom{N_A N_B}{k}}{\sqrt{k\eta}} Q\left(\sqrt{\frac{2}{\eta\lambda_s} + \frac{\beta}{k\eta}}\right) e^{\frac{1}{\eta\lambda_s} + \frac{\beta}{2k\eta}}, \quad (27)$$

and

$$\overline{SER}_{BA} = \frac{\alpha\sqrt{\beta\pi}}{\sqrt{2}} \sum_{k=1}^{N_A+N_B-1} (G_1 + G_2), \quad (28)$$

where

$$G_1 = \sum_{l=N_A N_B - k}^{N_A N_B - 1} \sum_{m=0}^l \frac{(-1)^m \mu_{k,l,m}}{\sqrt{(N_A N_B - l + m)\eta}} Q\left(\sqrt{\frac{2}{\eta\lambda_s} + \frac{\beta}{(N_A N_B - l + m)\eta}}\right) e^{\frac{1}{\eta\lambda_s} + \frac{\beta}{2(N_A N_B - l + m)\eta}}, \quad (29)$$

and

$$G_2 = \sum_{m=1}^{N_A N_B} \frac{(-1)^m \mu_{k, N_A N_B, m}}{\sqrt{m\eta}} Q\left(\sqrt{\frac{2}{\eta\lambda_s} + \frac{\beta}{m\eta}}\right) e^{\frac{1}{\eta\lambda_s} + \frac{\beta}{2m\eta}}. \quad (30)$$

Proof: The derivation is given in Appendix D. ■

The SER performance converges to an error floor, when the average SNR λ_s increases to infinity

$$\begin{aligned} \overline{SER}_{\lambda \rightarrow \infty}^{S-Max} \rightarrow & 1 - \frac{w\alpha\sqrt{\beta\pi}}{\sqrt{2}} \sum_{k=1}^{N_A N_B} \frac{(-1)^{k+1} \binom{N_A N_B}{k}}{\sqrt{k\eta}} Q\left(\sqrt{\frac{\beta}{k\eta}}\right) e^{\frac{\beta}{2k\eta}} \\ & - \frac{(1-w)\alpha\sqrt{\beta\pi}}{\sqrt{2}} \sum_{k=1}^{N_A+N_B-1} \frac{(-1)^{m+1} \mu_{k, N_A N_B, m}}{\sqrt{m\eta}} Q\left(\sqrt{\frac{\beta}{m\eta}}\right) e^{\frac{\beta}{2m\eta}} \\ & - \frac{(1-w)\alpha\sqrt{\beta\pi}}{\sqrt{2}} \sum_{k=1}^{N_A+N_B-1} \sum_{l=N_A N_B - k}^{N_A N_B} \sum_{m=0}^l \frac{(-1)^{m+1} \mu_{k,l,m}}{\sqrt{(N_A N_B - l + m)\eta}} \\ & \times Q\left(\sqrt{\frac{\beta}{(N_A N_B - l + m)\eta}}\right) e^{\frac{\beta}{2(N_A N_B - l + m)\eta}}. \end{aligned} \quad (31)$$

On the other hand, when $\eta = 0$, i.e. the self interference is perfectly canceled, we can further calculate the asymptotic SER of the Serial-Max method at high SNR. Firstly, the CDF expression of γ_{AB} can be rewritten as

$$F_{\gamma_{AB}}(x) = \left(1 - e^{-\frac{x}{\lambda_s}}\right)^{N_A N_B} \quad (32)$$

Then, the first order expansion of its corresponding PDF is given by

$$f_{\gamma_{AB}}(x) = N_A N_B \frac{x^{N_A N_B - 1}}{\lambda_s^{N_A N_B}} + o\left(x^{N_A N_B - 1 + \varepsilon}\right). \quad (33)$$

Using (25), the asymptotic SER of γ_{AB} can be obtained

$$\overline{SER}_{AB} = \frac{u_1}{\lambda_s^{N_A N_B}} + o\left(\frac{1}{\lambda_s^{N_A N_B}}\right), \quad (34)$$

where

$$u_1 = \frac{2^{N_A N_B - 1} \alpha \Gamma\left(N_A N_B + \frac{1}{2}\right)}{\beta^{N_A N_B} \sqrt{\pi}}. \quad (35)$$

Similarly, when the self interference cancelation is perfect, the CDF of γ_{BA} is rewritten as

$$F_{\gamma_{BA}}(x) = \sum_{k=1}^{N_A + N_B - 1} \sum_{l=N_A N_B - k}^{N_A N_B} \mu'_{k,l} \left(1 - e^{-\frac{x}{\lambda_s}}\right)^l e^{-\frac{(N_A N_B - l)x}{\lambda_s}}, \quad (36)$$

where

$$\mu'_{k,l} = \frac{\binom{N_A N_B - k - 1}{N_A + N_B - k - 1} \binom{N_A N_B}{l}}{\binom{N_A N_B - 1}{N_A + N_B - 2}}. \quad (37)$$

Then, the first order expansion of its corresponding PDF is calculated as

$$f_{\gamma_{BA}}(x) = \mu'_{N_A + N_B - 1, (N_A - 1)(N_B - 1)} (N_A - 1)(N_B - 1) \frac{x^{(N_A - 1)(N_B - 1) - 1}}{\lambda_s^{(N_A - 1)(N_B - 1)}} + o\left(x^{(N_A - 1)(N_B - 1) + \varepsilon}\right) \quad (38)$$

Combing (25), the asymptotic SER of γ_{BA} is

$$\overline{SER}_{BA} = \frac{u_2}{\lambda_s^{(N_A - 1)(N_B - 1)}} + o\left(\frac{1}{\lambda_s^{(N_A - 1)(N_B - 1)}}\right), \quad (39)$$

where

$$u_2 = \frac{2^{N_A N_B - N_A - N_B} \alpha \Gamma\left((N_A - 1)(N_B - 1) + \frac{1}{2}\right) \binom{N_A N_B}{(N_A - 1)(N_B - 1)}}{\beta^{(N_A - 1)(N_B - 1)} \sqrt{\pi} \binom{N_A N_B - 1}{N_A + N_B - 2}}. \quad (40)$$

Combining (34) and (39), the asymptotic weighted sum SER with perfect interference cancelation is expressed as

$$\overline{SER}^{S-Max} = \frac{(1 - w)u_2}{\lambda_s^{(N_A - 1)(N_B - 1)}} + o\left(\frac{1}{\lambda_s^{(N_A - 1)(N_B - 1)}}\right). \quad (41)$$

It is implied by (41) that given $0 < w < 1$ the diversity order of the Serial-Max method is $(N_A - 1)(N_B - 1)$ with $\eta = 0$. This is coincident with a simple deduction of the existing result [27]: The diversity order is determined by the worse link, i.e., the second best link which is selected from Γ' consisting of $(N_A - 1)(N_B - 1)$ i.i.d. elements. Moreover, the transmission direction with the greater weight will achieve the full diversity order of $N_A N_B$, and the other direction will achieve the diversity order of $(N_A - 1)(N_B - 1)$ for $w \neq 0.5$. On the condition that $w = 0.5$, α can be a arbitrary fraction. Then the best link can be arbitrarily allocated to each direction. Therefore, the diversity orders of the two directions are both $(N_A - 1)(N_B - 1)$, and the achievable diversity is obviously $(N_A - 1)(N_B - 1)$ for the weighted sum SER.

V. SIMULATION RESULTS

In this section, we provide the simulation results for our proposed SBLS methods to validate the previous analysis. For simplicity, we consider that $N_A = N_B = N$ in the following simulations.

A. Average Weighted Sum Rate

Fig. 2 depicts the average weighted sum rate of the Max-WSR and Serial-Max methods versus SNR with different levels of self interference $\eta = 0.02, 0.05, 0.1$ for $N = 3$ and $w = 0.7$. It can be seen that the weighted sum rate expression in (17) perfectly matches with the simulation results. In addition, the weighted sum rate performance is limited by rate ceilings, which coincides with the preceding analysis in (22). From the figure, we find that at low λ_s , the weighted sum rate performance for different η is quite similar, because the weighted sum rate performance at low λ_s is SNR-limited. However, at large λ_s , the residual self interference will dominate the performance, and the performance is limited by the rate ceiling caused by the residual self interference. The figure also reveals that the Serial-Max method achieves almost the same average weighted sum rate as the Max-WSR method across all SNR regions.

In Fig. 3, we illustrate the average weighted sum rate of the Max-WSR and Serial-Max methods for different numbers of antennas $N = 3, 4, 5$ where the self interference cancellation coefficient is $\eta = 0.02$. It can be observed that, for different numbers of antennas, the Serial-Max algorithm achieves almost the same average weighted sum rate as the Max-WSR one across all SNR region. We can also find from this figure that the average weighted sum rate increases with the number of antennas.

B. Average Weighted Sum SER

The following simulations of weighted sum SER performance are conducted with BPSK modulation. In Fig. 4, the weighted sum SER performance of the Serial-Max method is provided for different $\eta = 0, 0.05, 0.1, 0.5$, where large η means severe self interference whereas small η means slight self interference level. Especially, $\eta = 0$ means perfect interference cancellation. This figure verifies the weighted sum SER expression given by Proposition 3. Based on the figure, it can be observed that the simulated SER performance for $\eta = 0$ tightly matches with the asymptotic one given by (41) at high SNR, while the SER performances for $\eta > 0$ are constrained

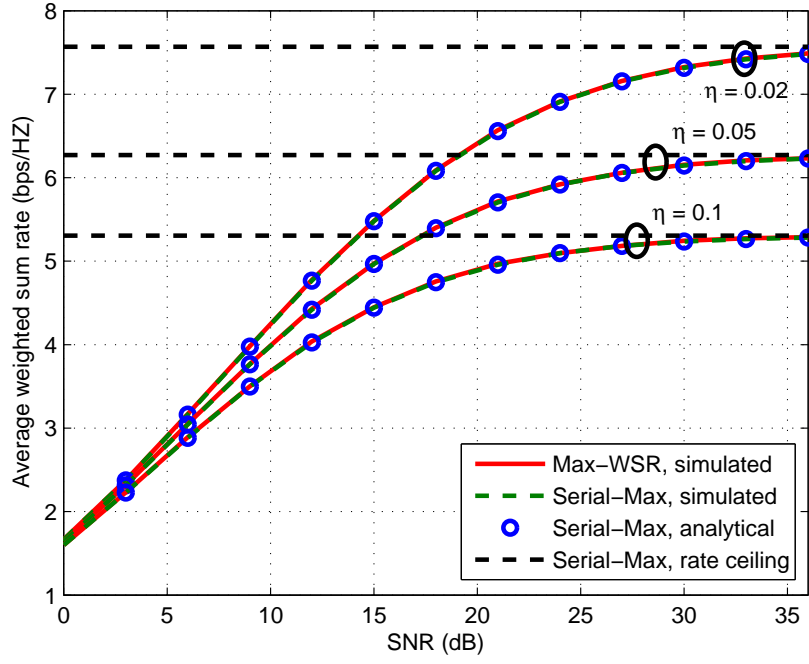


Fig. 2. Weighted sum rate performances of the Max-WSR and Serial-Max methods, where $w = 0.7$, $N = 3$ and $\eta = 0.02, 0.05, 0.1$.

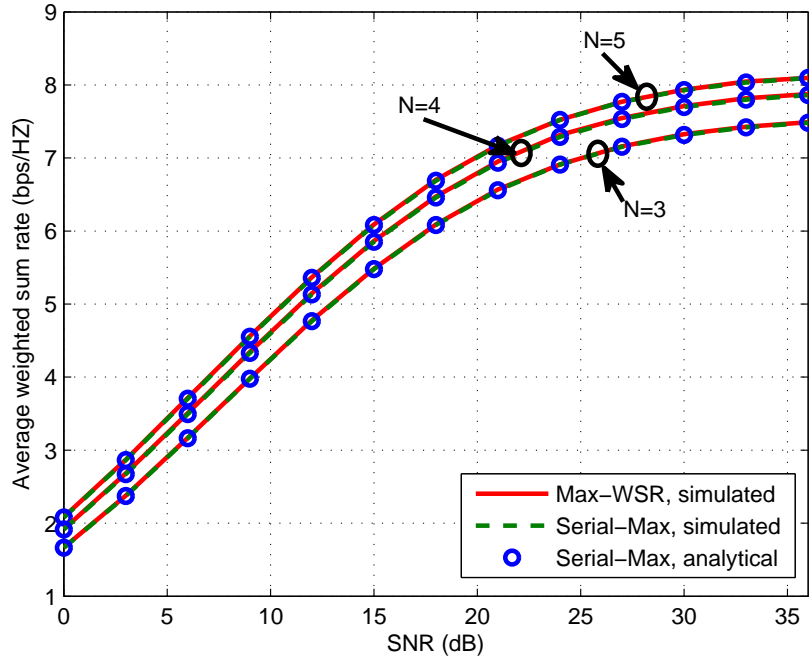


Fig. 3. Weighted sum rate performances of the Max-WSR and Serial-Max methods, where $w = 0.7$, $\eta = 0.02$ and $N = 3, 4, 5$.

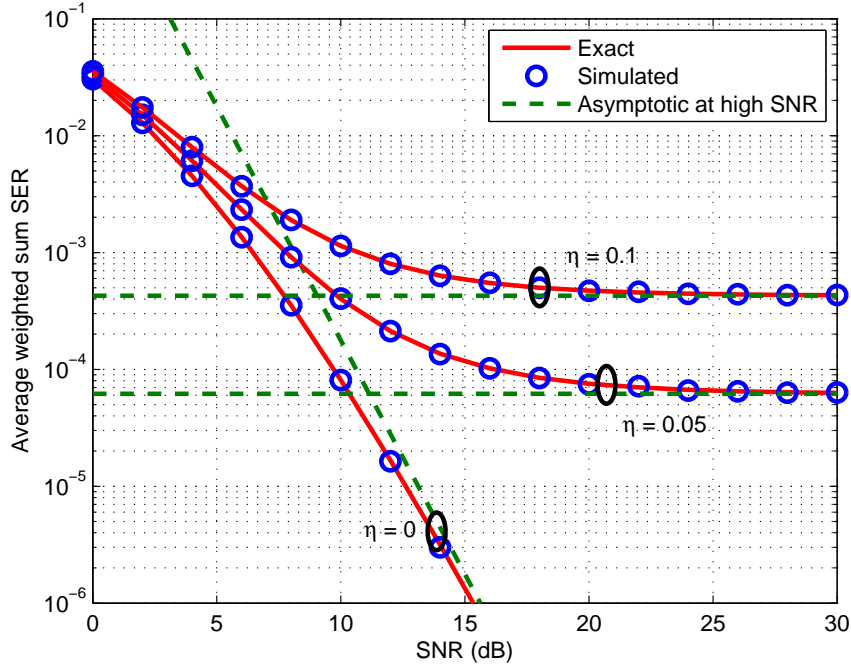


Fig. 4. Weighted sum SER performances of the Serial-Max method, where $N = 3$ and $\eta = 0, 0.05, 0.1, 0.5$.

by error floors evaluated by (31) at high SNR. It can also be seen that when the self interference is perfectly canceled, the diversity order of the Serial-Max method is $(N_A - 1)(N_B - 1)$. However when the residual self interference exists, the Serial-Max method has a zero diversity order.

Fig. 5 shows the average weighted sum SER for different numbers of antennas $N = 3, 4, 5$ where the self interference level $\eta = 0.1$ is assumed. It shows that both the Min-WSER and Serial-Max methods are limited by error floors at high SNR due to the residual self interference. As the number of antennas increase, the SER performances of both methods including the error floor are improved. Moreover, the Serial-Max method performs closer to the Min-WSER scheme as N increases.

Fig. 6 compares the simulated weighted sum SER performance of the Min-WSER and Serial-Max methods with different number of antennas. Combinations of different SNR and self interference levels ($\lambda_s = 10, 15\text{dB}$, $\eta = 0.1, 0.2$) are provided. It can be observed that the gaps between the Min-WSER and Serial-Max methods are reduced as N increases. It also shows that the SER performance of the Serial-Max method approaches the Min-WSER method when

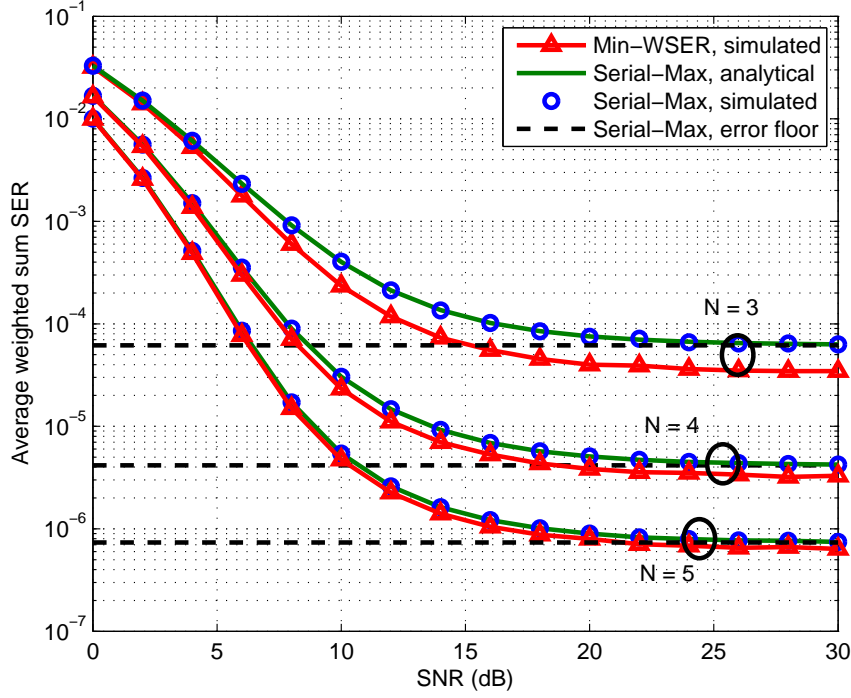


Fig. 5. Weighted sum SER performances of the Min-WSER and Serial-Max methods, where $\eta = 0.05$ and $N = 3, 4, 5$.

the self interference is large or SNR is small. This is because in these cases these two factors dominate the SER performances of both methods.

C. Computational complexity

To compare the computational complexity of the optimal and Serial-Max methods, the number of required floating-point operations (flops) are presented in Fig. 7. It is clear that the optimal method has a high complexity due to the brute-force search operation, while the Serial-Max method can provide significant a complexity reduction, especially when the number of antennas is large.

VI. CONCLUSIONS

In this paper, we have proposed an opportunistic bidirectional link selection approach in FD MIMO systems. The optimal approach based on the “obtainable SINR”, defined as the ratio of the instantaneous SNR and average INR plus one, requires exhaustive search, bearing high

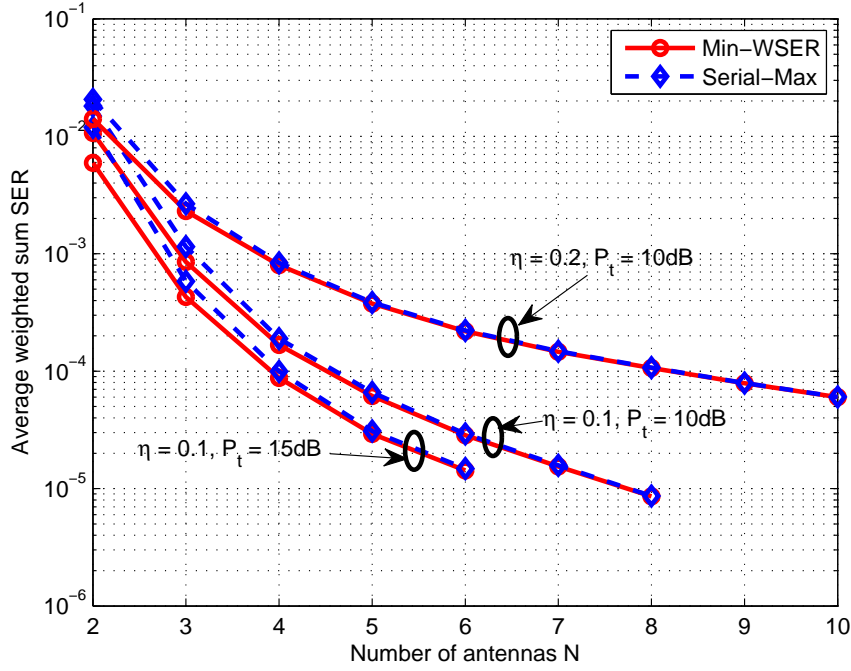


Fig. 6. Simulated weighted sum SER performances of the Min-WSER and Serial-Max methods, where $\eta = 0.1, 0.2$ and $P_t = 10, 15\text{dB}$.

complexity. To facilitate the selection process and reduce the computational complexity, a simple Serial-Max method with near optimal performances was proposed. The performance analysis was provide for the Serial-Max method, including the weighted sum rate and SER performances. It was shown that the proposed Serial-Max method approaches the respective weighted sum rate and weighted sum SER performances of the exhaustive search methods when the number of antennas increases.

APPENDIX A: PROOF OF LEMMA 2

According to the description of the Serial-Max method, the CDF of γ_{AB} is expressed as

$$\begin{aligned} F_{\gamma_{AB}}(x) &= \Pr \left(\frac{\gamma_s^{I_1 J_1}}{\gamma_B^{RI} + 1} < x \right) \\ &= \Pr \left(\gamma_s^{I_1 J_1} < x (\gamma_B^{RI} + 1) \right), \end{aligned} \quad (42)$$

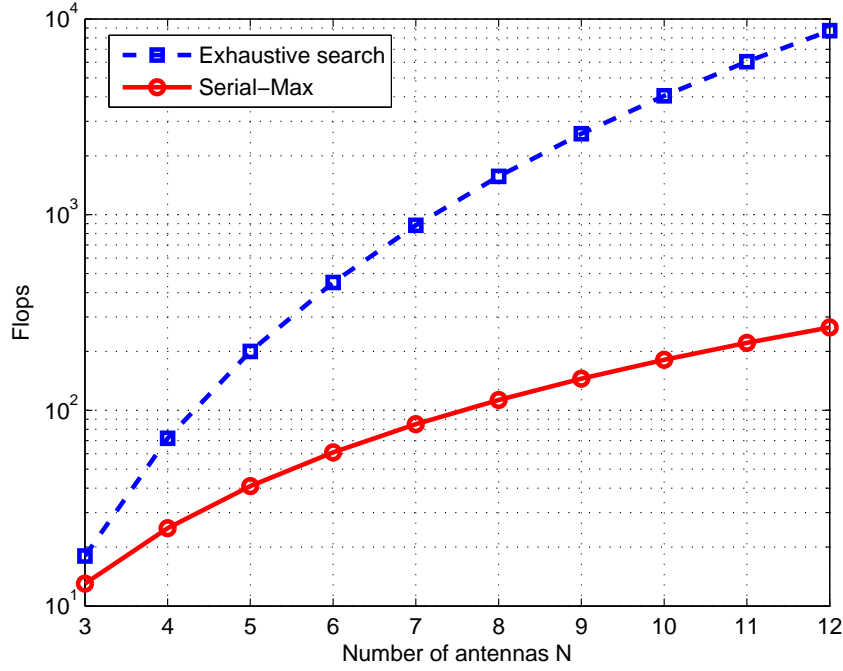


Fig. 7. Complexity comparison between the optimal method and the Serial-Max method.

where $\gamma_s^{I_1 J_1}$ is the largest one of $N_A \times N_B$ i.i.d. exponential-distributed random variables, and its CDF is given by [23]

$$F_{\gamma_s^{I_1 J_1}}(x) = \left(1 - e^{-\frac{x}{\lambda_s}}\right)^{N_A N_B}. \quad (43)$$

γ_B^{RI} follows exponential distribution with average λ_i . Substituting the CDF expression of $\gamma_s^{I_1 J_1}$ and γ_B^{RI} , (42) can be calculated as

$$F_{\gamma_{AB}}(x) = \int_0^\infty \left[1 - e^{-\frac{x(\gamma_B^{RI} + 1)}{\lambda_s}}\right]^{N_A N_B} \frac{e^{-\frac{\gamma_B^{RI}}{\lambda_i}}}{\lambda_i} d\gamma_B^{RI}. \quad (44)$$

Through some manipulations, we can obtain the CDF of γ_{AB} .

APPENDIX B: PROOF OF LEMMA 3

As aforementioned, we have $\gamma_{BA} = \gamma_s^{I_2 J_2} / (\gamma_A^{RI} + 1)$, and its CDF expression is

$$\begin{aligned} F_{\gamma_{BA}}(x) &= \Pr\left(\frac{\gamma_s^{I_2 J_2}}{\gamma_A^{RI} + 1} < x\right) \\ &= \Pr(\gamma_s^{I_2 J_2} < x(\gamma_A^{RI} + 1)). \end{aligned} \quad (45)$$

$\gamma_s^{I_2 J_2}$ is the maximum order statistic in the pruned matrix Γ'_s consisting of $(N_A - 1) \times (N_B - 1)$ random variables following the exponential distribution. Since Γ'_s is obtained by removing the maximum element and other $N_A + N_B - 2$ elements from Γ_s , $\gamma_s^{I_2 J_2}$ could possibly be one of any $(N_A N_B - k)$ -th order statistic of Γ_s , $k = \{1, 2, \dots, N_A + N_B - 1\}$. As a result, the CDF of $\gamma_s^{I_2 J_2}$ is given by

$$F_{\gamma_s^{I_2 J_2}}(x) = \sum_{k=1}^{N_A + N_B - 1} p_k F_{\gamma}^{(N_A N_B - k)}(x), \quad (46)$$

where p_k is the probability that $\gamma_s^{I_2 J_2}$ is the $(N_A N_B - k)$ -th order statistic in Γ_s , and $F_{\gamma}^{(N_A N_B - k)}(x)$ represents the CDF of the $(N_A N_B - k)$ -th order statistic among $N_A N_B$ variables.

For n i.i.d. variables, each with CDF $F(x)$, the CDF of the r -th largest order statistic can be written as

$$F^{(r)}(x) = \sum_{i=r}^n \binom{n}{i} F^i(x) [1 - F(x)]^{n-i}. \quad (47)$$

Here, they are all exponential variables with average λ_s . Then, the CDF of the $(N_A N_B - k)$ -th order statistic can be expressed as

$$\begin{aligned} F_{\gamma}^{(N_A N_B - k)}(x) &= \sum_{l=N_A N_B - k}^{N_A N_B} \binom{N_A N_B}{l} \left(1 - e^{-\frac{x}{\gamma}}\right)^l e^{-\frac{N_A N_B - l}{\gamma} x} \\ &= \sum_{l=N_A N_B - k}^{N_A N_B} \sum_{m=0}^l \binom{N_A N_B}{l} \binom{l}{m} (-1)^m e^{-\frac{N_A N_B - l + m}{\gamma} x}. \end{aligned} \quad (48)$$

On the other hand, if $\gamma_s^{I_2 J_2}$ is the $(N_A N_B - k)$ -th order statistic in Γ_s , it means that among the $(N_A + N_B - 1)$ elements removed from the matrix Γ_s , there are k elements larger than $\gamma_s^{I_2 J_2}$. To satisfy that the largest element of Γ'_s is the $(k + 1)$ largest element in Γ_s , we can first remove the first k largest elements in the matrix Γ_s , and then, keeping the $(k + 1)$ -th largest element unremoved, we remove the other $(N_A + N_B - 1 - k)$ elements randomly. Thus, p_k is calculated as

$$p_k = \frac{\binom{N_A N_B - k - 1}{N_A + N_B - k - 1}}{\binom{N_A N_B - 1}{N_A + N_B - 2}}, \quad (49)$$

where the numerator implies that when we remove $(N_A + N_B - 1 - k)$ elements randomly from $(N_A N_B - k - 1)$ elements, there are $\binom{N_A N_B - k - 1}{N_A + N_B - k - 1}$ possibilities, while the denominator means that when we remove $(N_A + N_B - 2)$ elements (besides $\gamma_s^{I_1 J_1}$, the largest one of Γ_s) randomly from $(N_A N_B - 1)$ elements, there are $\binom{N_A N_B - 1}{N_A + N_B - 2}$ possibilities.

Finally, substituting (48) and (49) into (46), the CDF expression of $\gamma_s^{I_2 J_2}$ can be obtained as

$$F_{\gamma_s^{I_2 J_2}}(x) = \sum_{k=1}^{N_A+N_B-1} \sum_{l=N_A N_B-k}^{N_A N_B} \sum_{m=0}^l (-1)^m \mu_{k,l,m} e^{-\frac{N_A N_B-l+m}{\lambda} x}, \quad (50)$$

where $\mu_{k,l,m}$ is expressed as

$$\mu_{k,l,m} = \frac{\binom{N_A N_B-k-1}{N_A+N_B-k-1} \binom{N_A N_B}{l} \binom{l}{m}}{\binom{N_A N_B-1}{N_A+N_B-2}}. \quad (51)$$

Then, substituting (50) into (45) and combining the CDF of γ_A^{RI} we have

$$F_{\gamma_{BA}}(x) = \int_0^\infty \sum_{k=1}^{N_A+N_B-1} \sum_{l=N_A N_B-k}^{N_A N_B} \sum_{m=0}^l (-1)^m u_{k,l,m} e^{-\frac{N_A N_B-l+m}{\lambda_i} x (\gamma_A^{RI} + 1)} \frac{e^{-\frac{\gamma_A^{RI}}{\lambda_i}}}{\lambda_i} d\gamma_A^{RI}. \quad (52)$$

Through some manipulations, Lemma 3 can be obtained.

APPENDIX C: PROOF OF PROPOSITION 2

Combining the CDF expression of γ_{AB} in (13) and (16), we can obtain

$$\begin{aligned} \bar{R}_{AB} &= \frac{1}{\ln 2} \sum_{k=1}^{N_A N_B} (-1)^{k+1} \binom{N_A N_B}{k} \int_0^\infty \frac{e^{-\frac{k}{\lambda_s} x}}{(k\eta x + 1)(1+x)} dx \\ &= \frac{1}{\ln 2} \sum_{k=1}^{N_A N_B} (-1)^{k+1} \binom{N_A N_B}{k} \int_0^\infty \frac{1}{1-k\eta} \left[\frac{e^{-\frac{k}{\lambda_s} x}}{(1+x)} - \frac{k\eta e^{-\frac{k}{\lambda_s} x}}{k\eta x + 1} \right] dx. \end{aligned} \quad (53)$$

Using the integral $E_1(x) = e^{-x} \int_0^\infty \frac{e^{-t}}{t+x} dt$ [22], (53) can be given by

$$\bar{R}_{AB} = \frac{1}{\ln 2} \sum_{k=1}^{N_A N_B} \frac{(-1)^k \binom{N_A N_B}{k}}{1-k\eta} \left[e^{\frac{1}{\eta \lambda_s}} E_1\left(\frac{1}{\eta \lambda_s}\right) - e^{\frac{k}{\lambda_s}} E_1\left(\frac{k}{\lambda_s}\right) \right] \quad (54)$$

On the other hand, the CDF of γ_{BA} in (14) can be rewritten as

$$F_{\gamma_{BA}}(x) = 1 - \sum_{k=1}^{N_A+N_B-1} (f_1 + f_2), \quad (55)$$

where

$$f_1 = \sum_{l=N_A N_B-k}^{N_A N_B-1} \sum_{m=0}^l (-1)^{m+1} \mu_{k,l,m} \frac{e^{-\frac{N_A N_B-l+m}{\lambda_s} x}}{(N_A N_B-l+m)\eta x + 1}, \quad (56)$$

and

$$f_2 = \sum_{m=1}^{N_A N_B} (-1)^{m+1} \mu_{k, N_A N_B, m} \frac{e^{-\frac{m}{\lambda_s} x}}{m\eta x + 1}. \quad (57)$$

Then, substituting (55)–(57) into (16), we have

$$\bar{R}_{BA} = \frac{1}{\ln 2} \sum_{k=1}^{N_A+N_B-1} \left(\int_0^\infty \frac{f_1 + f_2}{1+x} dx \right). \quad (58)$$

After some manipulations, the average rate \bar{R}_{BA} in (19) can be obtained. Finally, combining the expression of \bar{R}_{AB} and \bar{R}_{BA} , Proposition 2 can be proved.

APPENDIX D: PROOF OF PROPOSITION 3

Firstly, substituting the CDF of γ_{AB} in (13) into (23), we have

$$\overline{SER}_{AB} = \frac{1}{2} - \frac{\alpha\sqrt{\beta}}{2\sqrt{2\pi}} \sum_{k=1}^{N_A N_B} (-1)^{k+1} \binom{N_A N_B}{k} \int_0^\infty \frac{e^{-(\frac{\beta}{2} + \frac{k}{\lambda_s})x}}{(k\eta x + 1)\sqrt{x}} dx. \quad (59)$$

Applying Integration by substitution, the integral part is rewritten as

$$\int_0^\infty \frac{e^{-(\frac{\beta}{2} + \frac{k}{\lambda_s})x}}{(k\eta x + 1)\sqrt{x}} dx \stackrel{t=\sqrt{x}}{=} \int_0^\infty \frac{2e^{-(\frac{\beta}{2} + \frac{k}{\lambda_s})t^2}}{(k\eta t^2 + 1)t} dt. \quad (60)$$

Using the integral $\int_0^\infty \frac{e^{-a^2 x^2}}{x^2 + b^2} dx = Q(\sqrt{2}ab) \frac{\pi}{b} e^{a^2 b^2}$ [28], (60) can be calculated as

$$\int_0^\infty \frac{e^{-(\frac{\beta}{2} + \frac{k}{\lambda_s})x}}{(k\eta x + 1)\sqrt{x}} dx = \frac{2\pi}{\sqrt{k\eta}} Q\left(\sqrt{\frac{2}{\eta\lambda_s} + \frac{\beta}{k\eta}}\right) e^{\frac{1}{\eta\lambda_s} + \frac{\beta}{2k\eta}}. \quad (61)$$

Substituting (61) into (59), and after some manipulation, (27) can be obtained.

Then, the average SER of γ_{BA} in (28) can be obtained by substituting (14) into (23) and through the similar manipulations. Finally, combining the two average SER expressions, Proposition 3 can be proved.

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